Freezing of dynamical exponents in low dimensional random media

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A particle in a random potential with logarithmic correlations in dimensions $d = 1, 2$ is shown to undergo a dynamical transition at $T_{dyn} > 0$. In $d = 1$ exact results demonstrate that $T_{dyn} = T_c$, the static glass transition temperature, and that the dynamical exponent changes from $z(T) = 2 + 2(T_c/T)^2$ at high temperature to $z(T) = 4T_c/T$ in the glass phase. The same formulae are argued to hold in $d = 2$. Dynamical freezing is also predicted in the 2D random gauge XY model and related systems. In $d = 1$ a mapping between dynamics and statics is unveiled and freezing involves barriers as well as valleys. Anomalous scaling occurs in the creep dynamics.

The motion of topological defects by thermal activation over pinning barriers determines the slow glassy dynamics in numerous disordered systems, e.g. domain walls in dirty magnets, vortices in superconductors, dislocations in pinned lattices [1]. While treating many interacting extended defects in the presence of disorder remains a major challenge, progress can be made on the simpler, already non trivial problem of a single point defect. A model of great interest is a particle diffusing in a random potential with log-correlations, i.e barriers growing logarithmically with scale. In 2D it precisely describes a single vortex in a XY spin model with gaussian random gauge disorder [2], and is relevant to a host of related systems e.g. vacancies in pancake lattices of layered 3D superconductors [3,1], dislocations in 2D lattices with smooth disorder [4], electrons on helium [5]. As a prototype model of diffusion in a complex phase space or in random media, it is of broader interest to relaxation in glasses [6], transport in solids [7] population biology [8], non hermitian quantum mechanics [9] and vortex glass dynamical scaling [10]. Similar models where used to study the dynamical generation of broad (e.g. power law) distributions of relaxation times and its relation to aging [11]. Its static limit also appears in Integer Quantum Hall (QH) transition studies [12] and its quantum extension in QH bilayer systems [13].

Early studies of this model, in the context of tracer diffusion in 2d potential flows, used RG methods perturbative in the disorder strength σ [14–16]. The position $x(t)$ of a particle (in any d) satisfies a Langevin equation:

$$\dot{x}(t) = -\nabla V(x(t)) + \eta(t) \tag{1}$$

$\langle\eta(t)\eta(t')\rangle = 2T\delta(t-t')$ being a thermal noise ($\overline{X}$ and $\langle X\rangle$ respectively denote disorder and thermal averages). It was found that if correlations grow logarithmically:

$$\Delta(x-x') = \overline{(V(x)-V(x'))^2} \sim 4\sigma \ln|x-x'| \tag{2}$$

the diffusion is anomalous with a temperature T dependent "dynamical exponent" z given by [15,16]:

$$\overline{\langle x(t)^2\rangle} \sim t^{2/z} \quad , \quad z = 2 + 2\frac{\sigma}{dT^2} \tag{3}$$

Strikingly, this result for z was conjectured [15] to hold to *all orders* in $g = \sigma/dT^2$. This was confirmed in $d = 1$ and $d = 2$ by general arguments and a three loop calculation [17] (which also found $O(g^3)$ corrections to z in $d \geq 3$). Further support came from excellent agreement with simulations [18], performed in $d = 2$ for $0 < g < 0.8$, and exact results in $d = 1$ [10] for the velocity $v \sim f^{z-1}$ at small applied force f, with z again as in (3).

Recently, however, the *statics* of this model has been investigated in several works in $d = 2$ [2,12,19–21] and any d [22]. As a result we now know that there is in fact *a transition* at $T = T_c = \sqrt{\sigma/d}$ to a strong disorder, low T glass phase. This glass phase appears non trivial, as dominated by *a few* states, reminiscent of replica symmetry breaking (RSB). It is related, approximately, to the Random Energy Model (REM) [23] and, more closely, to the directed polymer on a Cayley tree (DPCT) [24]. It is thus an outstanding problem to investigate whether this equilibrium transition has a dynamical counterpart, and how it can be compatible with (3).

In this Letter we solve the apparent paradox. In $d = 1$ we demonstrate that there is indeed a dynamical transition at the static T_c and obtain the dynamical exponent $z(T)$ at all T. This is achieved through exact results and a real space RG (RSRG) method. Simple arguments and bounds also show that a dynamical transition occurs in $d = 2$, with analogous behaviour of $z(T)$. A similar dynamical freezing is also predicted in the 2d random gauge XY model, since the XY phase and its boundary are dominated by the single (i.e dilute) vortex limit [2,22].

We first give a simple argument indicating that the $1/T^2$ divergence in (3) *cannot* hold at low temperature. First, *in any given sample* (in finite d), the characteristic time t for a particle to escape a region of size L satisfies:

$$\ln t \approx B/T \leq [V_{max}(L) - V_{min}(L)]/T \equiv B_{max}/T \tag{4}$$

i.e is given as $T \to 0$ by the Arrhenius estimate where B is the energy barrier encountered by the particle, obviously bounded by the difference B_{max} between the absolute maximum $V_{max}(L)$ and minimum $V_{min}(L)$ of the potential in the region. Second, we now know [20,12,21,22] that these extremal values satisfy $V_{min}(L) = -2\sqrt{\sigma/d}(d\ln L - \frac{1}{2}\gamma\ln\ln L) + \delta V$ where

γ is a universal number and δV has $O(1)$ sample to sample fluctuations. Thus, defining $z \equiv \ln t/\ln L$ at large L [25] this yields, in any dimension d, the bound:

$$z \leq 4\sqrt{\sigma d}/T \qquad T \to 0. \tag{5}$$

Since (3) is exact perturbatively to all orders in $g = \sigma/T^2$ in $d = 1, 2$, (5) *implies* that a dynamical transition *must* occur in $d = 1, 2$ at a finite temperature $T_{dyn}(d) > 0$.

In $d = 1$, which we now study, it is natural to guess that the upper bound in (5) gives the exact $z(T)$ for all $T < T_c$. Indeed one expects that the Arrhenius law (4) should hold in the energy dominated glass phase and furthermore B should equal its upper bound in (4) since in $d = 1$ there is only a single path. We now confirm this with analytical results at all T using the first passage time approach [26]. We also show that (i) in $d = 1$ there is a direct correspondence between dynamical (e.g. exponents) and static quantities (ii) the dynamical exponent z can be unambiguously defined. Our conclusions being independent of boundary conditions we choose them reflecting at $x = 0$ [26] and define t as the first passage time at site $x = L$ of a particle which starts at $x = 0$. Some of our results are valid for *any* potential landscape but are mostly applied to gaussian log-correlated potentials (2).

As shown below it is sufficient to focus, in any given disorder realization $V(x)$, on the (thermal) *mean first passage time* $t_1(\{V\}) \equiv \langle t\rangle \equiv \tau$. We first obtain its *typical* behaviour (i.e in any sample), its large deviations being studied later. The exact formula for $t_1(\{V\})$ is [26]:

$$t_1(\{V\}) = \frac{1}{T}\int_0^L dy \int_0^L dx\ \theta(y-x)\ \mathrm{e}^{[V(y)-V(x)]/T}, \tag{6}$$

where $\theta(x)$ is the step function. Since the canonical partition function for the same disorder realization is $Z_L(\{V\}) \equiv \int_0^L dx\ \ \exp(-V(x)/T)$, (6) strongly resembles *two copies of the statics*, one with disorder $+V(x)$ (dominated by minima at low T), the other with $-V(x)$ (dominated by maxima of V at low T). Dynamics and statics are thus directly connected by: $\ln\{T[t_1(\{V\}) + t_1(\{-V\})]\} = \ln Z_L(\{V\}) + \ln Z_L(\{-V\})$ and also

$$\begin{aligned}\ln Z_{L/2}(\{-V\}) + \ln Z_{L/2}(\{V\}) &\leq \ln\{T t_1(\{V\})\} \\ &\leq \ln Z_L(\{-V\}) + \ln Z_L(\{V\}).\end{aligned} \tag{7}$$

If the ratio of upper to lower bound converges to unity in the thermodynamic limit, they uniquely determine the long time behavior in terms of the statics. This is the case for gaussian potentials with correlations growing at the most as a power of the logarithm of the scale. If the correlations do not grow faster than logarithmically with scale, then $\ln Z_L(\{V\}) \sim \ln L$. If, additionally, the intensive free energy $f = F_L(\{V\})/\ln L = -T\ln Z_L(\{V\})/\ln L$ is self-averaging, the dynamical exponent z is obtained as:

$$z(\{V\}) \equiv \lim_{L\to\infty} \ln t_1(\{V\})/\ln L = -2f/T, \tag{8}$$

and is also self averaging. This is the case both for uncorrelated and log-correlated gaussian potentials. For the former (8) gives the normal diffusion value $z = 2$. For the latter, using the known results for f [12,22] we obtain our main result for the dynamical exponent:

$$\begin{aligned} z(T) &= z_A(T) \equiv 2\left(1 + \sigma/T^2\right) && (\text{for } T \geq T_\mathrm{c}) \\ z(T) &= 4\sqrt{\sigma}/T && (\text{for } T \leq T_\mathrm{c}). \end{aligned} \tag{9}$$

Thus a dynamical transition, away from the "annealed" value $z_A(T)$ given by (3), occurs at the same temperature $T_{dyn} = T_c = \sqrt{\sigma}$ as the equilibrium transition. At T_c freezing occurs in the thermal configurations which dominate $\langle t\rangle$, simultaneously around minima and maxima of the potential (i.e in the two copies), thus in the effective barrier. Interestingly, this transition coincides with the onset of logarithmic corrections. Indeed one can also characterize *typical* finite size fluctuations. Using that $F_L = f(\ln L - \frac{1}{2}\gamma(T)\ln(\ln L)) + \delta F$ where δF has $O(1)$ sample to sample fluctuations [22] we find that $t_1(\{V\})/\tau_{typ}$ has a well defined $O(1)$ limit distribution at large L, the *typical* mean first passage time being $\tau_{typ} = L^{z(T)}(\ln L)^{-\alpha(T)}$ with $\alpha(T) = 2\gamma(T)\sqrt{\sigma}/T$, $\gamma(T) = 0$ for $T > T_c$ but $\gamma(T_c) = \frac{1}{2}$ and $\gamma(T) = \frac{3}{2}$ for $T < T_c$. For faster growing (e.g. power law) correlations, the ratio between the bounds in Eq. (7) does not converge to one, and even the leading order of $\ln t_1(\{V\})$ still fluctuates at large L [26] as in the unbiased Sinai model [27].

We now show as promised that z can be defined unambiguously from the *mean* first passage time alone. In principle one could define a full set of dynamical exponents $z_p(\{V\}) = \lim_{L\to\infty} \ln t_p(\{V\})/p\ln L$ from higher thermal moments $t_p(\{V\}) \equiv \langle t^p\rangle$. Using the expression [26]:

$$t_p(\{V\}) = \frac{p!}{T^p}\int_{0<x_i,y_i<L} \prod_{i=1}^{p} \theta_{y_i,x_i}\theta_{y_i,x_{i-1}} \mathrm{e}^{\frac{V(y_i)-V(x_{i-1})}{T}}$$

and $\theta_{x,y} \equiv \theta(x-y) \leq 1$ we obtain, comparing with (6), $\ln t_p(\{V\})/p\ln L \leq \ln t_1(\{V\})/\ln L + \ln p!/p\ln L$, which together with the general inequality $\langle t\rangle^p \leq \langle t^p\rangle$ leads to:

$$z_p(\{V\}) = z_1(\{V\}) \qquad \text{and} \qquad \overline{z_p} = \overline{z_1}. \tag{10}$$

for any integer $p \geq 1$. This is because the thermal distribution of t has exponential decay and all moments $p \geq 1$ are controlled at large L by the *largest* relaxation time in a given sample. Thus $z = z_1$ characterizes the dynamics.

By contrast, the distribution of escape times $P_L(\tau)$ with disorder realization has broad tails extending in the region $\tau \gg \tau_{typ} \sim L^{z(T)}$, where we obtained the estimate:

$$P_L(\tau)d\tau \approx \frac{d\tau}{\tau}\left(\frac{\tau_{typ}}{\tau}\right)^{\mu} \exp\left(-\mu^2\frac{\ln^2(\tau/\tau_{typ})}{8\ln L}\right). \tag{11}$$

Here $\mu = T/T_c$ and (11) is valid for (i) $T < T_c$ and (ii) for $T > T_c$ and $\tilde{z} \equiv \ln\tau/\ln L \geq 4$. For $T > T_c$ and

$z(T) < \tilde{z} < 4$ one has simply $P_L(\tau) \approx \tau_{typ}^{-1}(\tau_{typ}/\tau)^{1+\mu^2}$. This corresponds to a quadratic (plus linear) multifractal spectrum for rare occurrences of $\tilde{z}$. (11) can be obtained by a Kosterlitz RG analysis of (6), as in [22]. The result is a non linear Kolmogorov equation for $P_L(\tau)$ as a function of $\ln L$, identical to the one describing the partition sum of two directed polymers on a Cayley tree, seeing opposite disorder, with constrained endpoints $x < y$. It yields (11) up to log corrections, neglected here. More empirically (11) can be obtained from the moments:

$$\overline{\langle t\rangle^p} = \frac{1}{T^p}\int_0^L \prod_{i=1}^{p} [dy_i dx_i \theta(y_i - x_i)] \exp\Big\{ \sum_{i,j=1}^{p} \big[\Delta(y_i - x_j) - \frac{1}{2}\Delta(y_i - y_j) - \frac{1}{2}\Delta(x_i - x_j)\big]/2T^2 \Big\}. \quad (12)$$

which reads as a partition sum for $2p$ particles, p of type y (representing hills in the potential landscape) and p of type x (representing valleys). Same type particles attract via a potential $\Delta(r)/2T$ while those of opposite type repel via $-\Delta(r)/T$. This is similar to estimating $\overline{Z^p}$ in the statics, except that there only one kind of particles (representing valleys) appears. Here hills and valleys play symmetric roles. (12) can be estimated for log-correlated potentials and for integer $p \geq 1$ as follows. At high T an "entropic" variational saddle point dominates (with all y's and x's far away $O(L)$ from each other). At low T an "energetic" saddle point dominates (all y's close together within $O(1)$, all x's close together, y's and x's far away). This yields the large L behaviour (universal since unaffected by changes in $\Delta(r)$ for small r):

$$\begin{aligned} \overline{\langle t\rangle^p} \sim & \; L^{2p(1+\sigma/T^2)} && \text{for} \quad T \geq T_{\rm c,p}, \\ \sim & \; L^{2p(1/p+p\sigma/T^2)} && \text{for} \quad T < T_{\rm c,p}. \end{aligned} \quad (13)$$

i.e, as for $\overline{Z^p}$ in the statics (of this model [19,12], the REM and the DPCT) there is a sequence of transition temperatures $T_{\rm c,p} = \sqrt{p}\,T_{\rm c}$ for the moments. One can check via a saddle point calculation of $\overline{\langle t\rangle^p} = \int d\tau \tau^p P_L(\tau)$ that (13) is consistent with (11) and that the $T_{\rm c,p}$ correspond to a change of behaviour from rare events to typical events dominance as the saddle point crosses $\tilde{z}_{sp} = 4$. By analytically continuing $\overline{\langle t\rangle^p}$ as $p \to 0$ one can recover (9) and the transition at T_c in typical behaviour. The "entropic" saddle point still dominates for $T > T_c$, but the "energetic" saddle point is replaced at $T < T_c$ by a one-step RSB ansatz. Each kind of "particle" is arranged in p/m groups of m particles close by in space, while different groups are far away, with $m = T/T_c$ at the optimum, extending the static [21,22] and the REM and DPCT replica calculations [23]. Compared to conventional static RSB which only involves "valleys" the interesting feature here is a *nonequilibrium RSB* which also involves "hills": indeed, near degeneracies of distant barriers result, in the glass phase, in a *nonequilibrium* splitting of the thermal distribution of the diffusing particle into *a few* packets in a single environment [28]. These features are absent for weaker correlations (high T "entropic" saddle) or stronger ones [26], where an "energetic" saddle without RSB dominates (degeneracies are subdominant in the Sinai landscape except in the presence of a bias [27]).

If an external force is applied, the creep velocity v relates to the *disorder averaged* mean escape time of regions of size $L_0 \sim 1/f$ and is thus controlled by the annealed exponent $z_A(T)$, distinct from $z(T)$ at low T, a striking breakdown of naive dynamical scaling. Freezing manifests itself in large finite size corrections to v due to undersampling of the disorder average. From [10], $v^{-1} = \frac{1}{L}\langle t\rangle$ holds for $fL \gg 1$, where $\langle t\rangle$ is given by (6) in the tilted landscape $V(x) - fx$. It can be approximated as the average over N samples of sizes L_0 of the escape time in each sample, each distributed with $P_{L_0}(\tau)$. A saddle point estimate shows that below T_c, $v \sim f^{z_A - 1}$ for $y = -\ln L/\ln f > y^* = 2/\mu^2 - 1$, but that for smaller sizes $1 < y < y^*$, the typical $v \sim f^{z_m - 1}$ where the exponent $z_m = \frac{4T_c}{T}\sqrt{(1+y)/2} - y + 1$ smoothly interpolates between the annealed and quenched one.

The RSRG method, previously devised to describe diffusion in the Sinai landscape (for details see [27]) and in a broader class [29], allows to obtain complementary information, e.g. about distribution of positions. Here, in the log-correlated landscape, it is implemented numerically. From the original set of (i) alternating local extrema $V(x_i)$, (ii) their energy differences ("barriers" of heights $F_i = |V(x_i) - V(x_{i+1})|$) and (iii) the segments between them ("bonds" of lengths $\ell_i = x_{i+1} - x_i$), one constructs iteratively the "renormalized landscape at scale Γ" by removing as Γ increases all barriers between Γ and $\Gamma + d\Gamma$ and merging the corresponding bonds. This decimation retains only the large barriers and deep valleys. The Arrhenius dynamics of a particle starting at x_0 is

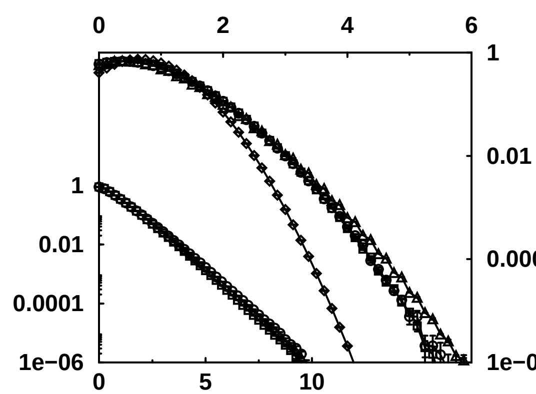

FIG. 1. Probability distributions for $\sigma = \frac{1}{2}$: (i) without and (ii) with additional short range disorder. Upper right: $Prob_\Gamma((F-\Gamma)/F^{\rm typ})$ for 2^{23} sites (i) with 2700 samples, initial ($\Gamma = 0$, triangles) and asymptotic ($\Gamma = 12$, squares) (ii) with 1000 samples initial ($\Gamma = 0$, diamonds) and asymptotic ($\Gamma = 18$, circles). Lower left: asymptotic $q(X = x/\overline{\ell}_\Gamma)$; (i) ($\Gamma = 12$, squares) (ii) ($\Gamma = 18$, circles).

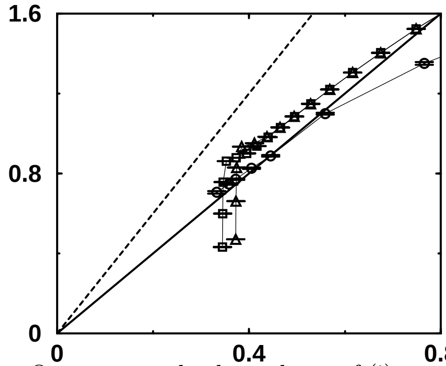

FIG. 2. On same graph: dependence of (i) maximum barrier $B_{max} = V_{max} - V_{min}$ on system size L (ii) minimum barrier $B = \Gamma$ left at decimation scale Γ on average bond length $\overline{\ell}_\Gamma$. Vertical axis: $4 - B/(\sqrt{\sigma}\ln R)$, horizontal axis: $\ln(4\pi \ln R)/\ln R$ with (i) $B = B_{max}$, $R = L$ (circles) and (ii) $B = \Gamma$, $R = \overline{\ell}_\Gamma$, $L = 2^{23}$ (squares) and $L = 2^{21}$ (triangles). A graph going to the origin means $B \sim 4\sqrt{\sigma}\ln R$ for $R \to \infty$. Finite size corrections $B = 4\sqrt{\sigma}(\ln R - \frac{1}{2}\gamma \ln(4\pi \ln R))$ with $\gamma = 3/2$ (dashed line) $\gamma = 1$ (full line) are shown.

approximated by putting it at time t at the bottom of the bond which contains x_0 in the renormalized landscape at $\Gamma = T\ln t$. The errors are small if the distribution of renormalized barriers is broad compared to T (infinitely broad in Sinai). Since here barriers remain finite, the method should be exact only as $T/T_c \to 0$, i.e when the thermal packet is concentrated in a single well. Perturbative corrections in T/T_c can be computed by considering the small occupation probability of neighboring secondary wells. In practice, the method works surprisingly better and gives the exact $z(T)$ up to T_c.

We have found that under renormalization the probability distributions for rescaled variables reach fixed point forms, namely $\mathrm{Prob}_\Gamma((F-\Gamma)/F^{\mathrm{typ}}_\Gamma) \to P^*((F-\Gamma)/F^{\mathrm{typ}})$ for rescaled barriers (Fig. 1) and $\mathrm{Prob}_\Gamma(\ell/\overline{\ell}_\Gamma) \to Q^*(\ell/\overline{\ell}_\Gamma)$. Here F^{typ}_Γ flows to a constant $F^{\mathrm{typ}} \approx 4.4\sqrt{\sigma}$ and $\overline{\ell}_\Gamma = L/N_\Gamma$ is the average bond length. Since two barriers are decimated at each step the number of remaining barriers N_Γ satisfies $\partial_\Gamma N_\Gamma = -2\alpha_\Gamma N_\Gamma$ where $\alpha_\Gamma \equiv Prob_\Gamma(F = \Gamma) \to \alpha^* = P^*(0)/F^{\mathrm{typ}}$, a constant. Thus the bond length grows as $\overline{\ell}_\Gamma \sim \exp(2\alpha^*\Gamma) \sim t^{1/z}$ and using $\Gamma = T\ln t$ one recovers the dynamical exponent $z = 1/(2\alpha^* T)$. Numerically we find $1/(2\alpha_\Gamma) \approx 4\sqrt{\sigma}(1 - 0.25\ln(4\pi \ln \overline{\ell}_\Gamma)/\ln \overline{\ell}_\Gamma)$ and thus a value of z consistent with (9). The diffusion front, computed as in [27], converges to a scaling form as $\overline{Prob_\Gamma(xt|00)} \to \overline{\ell}_\Gamma^{-1} q(X = x/\overline{\ell}_\Gamma)$, represented in Fig. 1. $\overline{\ell}_\Gamma$ is thus the only relevant lengthscale, all moments of the displacement scaling as $\overline{\langle x(t)^k \rangle} \sim \overline{\ell}_\Gamma^k \sim t^{k/z}$. Finally we obtained good numerical evidence (see Fig. 1) that all above asymptotic scaling functions, as well as $\alpha_0 \equiv \sqrt{\sigma}\alpha^*$ and $F_0 \equiv F^{\mathrm{typ}}/\sqrt{\sigma}$, do not change upon adding short-range disorder and are thus *universal* ($\overline{\ell}_\Gamma$ does change by a constant factor.)

We now address $d \geq 2$. First we note that the bound (5) can be improved to $z(T) \leq 2\sqrt{d\sigma}/T$ as $T \to 0$ for any $d \geq 2$. Indeed, in order to escape the particle now only needs to find a set of saddles which connects to the boundary. A percolation and counting argument shows that it can do so by remaining within sites such that $V(x)/\ln L \leq 0$. Thus the relevant barrier is bounded as $B \leq -V_{min}$. In $d = 2$ this bound is likely to be saturated since the particle still finds deepest minima, yielding $z = 2\sqrt{2\sigma}/T = 4T_c/T$. Since this expression matches (3) at the static T_c in $d = 2$, a likely scenario is that $T_{dyn} = T_c$ and that the expression holds for all $T < T_c$ [30]. A single vortex in a random gauge XY model will experience a similar dynamical freezing.

To conclude we demonstrated dynamical transitions in $d = 1, 2$. In $d = 1$ we found anomalous scaling of the creep velocity, novel freezing phenomena involving barriers and a finite T generalization of Arrhenius law $\tau \sim e^{-2F_L/T}$. Extensions will appear elsewhere.

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